# THE SHAPE FACTORS FOR A TRANSFORMATION FROM AREAL TO WEIGHT PARTICLES DISTRIBUTION

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## ABSTRACT

In many applications in the mining industry, the determination of several material features can only be performed if some image analysis method is utilized. Good examples of this are the size analysis of open pit muckpiles, the on-line size analysis of ores on a belt conveyor, the mineralogical composition analysis by microscopy, the lithological composition of ores, the on-line analysis of flotation froth, etc. In all cases the analysis is performed on a two dimensions basis (plane) and the information is required on a three dimensions (volumetric or weight) basis. In most cases then, areal particles distributions are obtained and usually a complex stereological transformation is used to obtain the required weight particles distributions.

In this work a simpler approach is presented. The required transformation is developed starting from the well-known transformation from surface to weight distribution and the Heywood shape factors equations. On these basis an areal to weight distribution transformation is proposed. The new procedure is experimentally tested using samples from seven different lithologies present in the Andina copper ore. Good results are obtained with this methodology. The variability in the shape factors due to lithology, particle size and amplitude of the class size is analyzed. Finally, the validity of the Heywood approach is discussed.

## INTRODUCTION

In the mining industry, the determination of several material features can only be adequately performed if some image analysis method is utilized.

Many applications are presented in the technical literature, such as the size analysis of open pit muckpiles (Smith and Kemeny, 1993), the on-line size analysis of ores on a belt conveyor (Lee and Herbst, 1992; Cartes et al., 1995), the mineralogical composition analysis by microscopy (Petruk, 1988; Oestreich et al., 1995), the lithological composition of ores (Casali et al., 2000), the on-line analysis of flotation froth (Guarini et al., 1995; Moolman et al., 1995), etc.

In all cases the analysis is performed on a two dimensions basis (plane) and the information is required on a three dimensions (volumetric or weight) basis. In most cases then, areal particles distributions are obtained and usually a complex stereological transformation is used to obtain the required weight particles distributions.

Research in stereology has led to several methods for the conversion of data obtained on sections or projected areas. These methods are usually applied for finding estimates of parameters (mean, variance and skewness) in three dimensions when two-dimensional data are observed. A good description of the basic concepts of stereology has been prepared by Barbery (Barbery, 1991a).

Most of the stereological methods are limited because their reliance on one or more restrictive assumptions such as presupposing a particular functional form for the particle size distribution and/or requiring particles being of regular geometrical form (cubic, spherical, ellipsoidal) and/or congruent and/or convex (King, 1982).

King proposed a new method, general and applicable to particles of irregular shape, which is free from the usual limiting assumptions. This method assume only that a statistical description of particle shape is invariant with particle size (King, 1982).

Lin et al. developed a reconstruction method from measurement on sections (Lin et al., 1987). Main drawbacks of this method are: the influence of particle shape on the results, the complexity and potential unstability of the mathematical procedure and the unknown effect of variations with particle size in the sample measured (Barbery, 1991a).

Barbery presents the development and validation of a new reliable method of reconstructing particle composition distribution from measurements on sections (Barbery, 1991b). This method release on a small number of well-defined assumptions: particle convexity and the relationship between particle volume and particle composition.

In this work, starting from the well-known transformation from surface to weight distribution and the Heywood shape factors equations; a simpler approach is developed.

## TRANSFORMATION METHOD

Starting from the procedure (Herbst, 1979) that allows the transformation from surface to weight distribution and considering the relation between surface and projected area, it can be shown that the transformation from areal (projected) distribution to weight distribution can be expressed as follows,

$$f_{3i} = \frac{\left(\frac{\alpha_V}{\alpha_S}\right)_i \left(\frac{S}{A}\right)_i \rho_i \, \overline{d}_i \, f_{2i}}{\sum_{j=1}^{n} \left(\frac{\alpha_V}{\alpha_S}\right)_j \left(\frac{S}{A}\right)_j \rho_j \, \overline{d}_j \, f_{2j}} \quad (1)$$

where:

$f_{3i}$ = mass fraction of particles belonging to class "i".

$\left(\frac{\alpha_V}{\alpha_S}\right)_i$ = ratio between volume and surface area shape factors, corresponding to particles belonging to class "i".

$\left(\frac{S}{A}\right)_i$ = ratio between surface and projected area of particles belonging to class "i".

$\rho_i$ = density of particles belonging to class "i".

$\overline{d}_i$ = areal weighted average diameter of particles belonging to class "i".

$f_{2i}$ = areal (projected) fraction of particles belonging to class "i".

$n$ = number of classes considered.

A rigorous analysis of shape factors has been given by Heywood (Kelly and Spottiswood, 1982), who considered the shape of the particle to have two distinct characteristics: the relative proportions of length, breadth, and thickness, and the geometric form. The relative proportions are evaluated by: elongation ratio, $R_E$ = length / breadth and flatness ratio, $R_F$ = breadth / thickness.

Heywood expresses the geometric form by two shape factors: the volume shape factor, $\alpha_V$, and the surface area shape factor, $\alpha_S$.

Both kinds of characteristics are related as follows,

$$\alpha_V = \frac{\alpha_{Vae}}{R_F \sqrt{R_E}} \quad (2)$$

$$\alpha_S = 1.57 + K_g \left(\frac{\alpha_{Vae}}{R_F}\right)^{4/3} \left(\frac{R_E + 1}{R_E}\right) \quad (3)$$

where:

$\alpha_{Vae}$ = equidimensional volume shape factor.

$K_g$ = constant that depends on the geometric form.

Using equations 2 and 3, it is possible to express the ratio between the volume and surface area shape factors as follows:

$$\left(\frac{\alpha_S}{\alpha_V}\right)_j = \frac{C_{1j} R_{Fj} \sqrt{R_{Ej}}}{(\alpha_{Vae})_j} + \frac{C_{2j} (\alpha_{Vae})_j^{1/3} (R_{Ej} + 1)}{(R_{Fj})^{1/3} + \sqrt{R_{Ej}}} \quad (4)$$

where $C_{1j}$ and $C_{2j}$ are constants to be adjusted for each class j. To determine $C_{1j}$ and $C_{2j}$ it is necessary to know $\alpha_V$, $\alpha_{Vae}$, $\alpha_S$, $R_E$ and $R_F$.

The volume shape factor, $\alpha_V$, can be determined dividing the volume by the cubic mean diameter of each particle. To allow this calculation it is necessary to measure the number, weight, mean diameter and density of a sample of particles of each class.

To calculate $R_E$ and $R_F$ it is necessary to measure directly, if it is possible, length, breadth and thickness of each particle. If the direct measurement is not possible, then the following estimates can be used: areal weighted average of maximum diameters ($d_{max}$) as length, areal weighted average of the mean perpendicular rope at each $d_{max}$ as breadth, and a mean thickness.

Knowing $\alpha_V$, $R_E$ and $R_F$, $\alpha_{Vae}$ can be determined using equation 2.

The surface area shape factor can be determined dividing the surface by the squared mean diameter of each particle. The surface (S) is difficult to measure, but it can be estimated from the projected area (A) as follows:

$$S = P_A \bar{e} + 2A \quad (5)$$

where:

$P_A$ = perimeter of the projected area (A).

$\bar{e}$ = mean thickness.

Using equations 2, 4 and 5, the transformation from areal (projected) distribution to weight distribution, expressed in equation 1, becomes possible.

## RESULTS

The new procedure is experimentally tested using samples from seven different lithologies present in the Andina copper ore. In a first approach, direct measurements of 10 rocks (one by one), from each lithological class, in the –15 +5 cm size class, were used to determine the components of the equation 1. The results are presented in Table I.

Table I. Experimental and Estimated Fractions

| Lithological Class | Mass Fraction $f_{3i}$ [%] | Areal Fraction $f_{2i}$ [%] | Estimated $f_{3i}$ [%] |
|---|---|---|---|
| Turmaline Brec. | 8.5 | 9.9 | 5.5 |
| Other Breccias | 17.2 | 17.9 | 15.2 |
| Porph. Dykes | 20.4 | 18.3 | 25.5 |
| Dac. Diatreme | 24.2 | 17.5 | 21.4 |
| Granodiorites | 13.3 | 14.1 | 12.4 |
| Andesite | 8.8 | 9.9 | 9.4 |
| Riol. Diatreme | 7.6 | 12.4 | 10.6 |
| TOTAL: | 100.0 | 100.0 | 100.0 |

As can be seen, good results are obtained with this methodology, but the question remains about what could happen if instead of using direct measurements (which are not always possible), other techniques, as image analysis, are used.

In a second approach, more realistic in terms of its applicability, the image of a previously known composite disposed randomly on a multi layer arrangement, was captured by a video camera and analyzed by image analysis software. The required measurements were made and the transformation method applied. The results are shown in Table II.

Table II: Experimental and Estimated Mass Fractions

| Lithological Class | Mass Fraction $f_{3i}$ [%] | Estimated $f_{3i}$ [%] | Squared Residue [%]$^2$ |
|---|---|---|---|
| Turmaline Brec. | 9.8 | 6.8 | 9.0 |
| Other Breccias | 17.0 | 14.8 | 4.8 |
| Porph. Dykes | 17.2 | 17.6 | 0.2 |
| Dac. Diatreme | 14.3 | 16.9 | 6.8 |
| Granodiorites | 14.5 | 13.5 | 1.0 |
| Andesite | 11.4 | 16.4 | 25.0 |
| Riol. Diatreme | 15.8 | 14.0 | 3.2 |
| TOTAL: | 100.0 | 100.0 | 50.0 |

The associated error can be estimated from the sum of the squared residues (SSR) as follows,

$$Error \approx \sqrt{\frac{SSR}{(n-1)}} \quad (6)$$

SSR = 50 and n (number of lithologies) = 7, then the error is lesser than 3%.

Because of its importance for the new method, the variabilities of the shape factors due to lithology, particle size and amplitude of the class size are analyzed. The variation with respect to lithology, is presented in Table III.

To test the variations with particle size, the shape factor was determined for another copper ore and for different granulometric classes, with particle sizes from 5 to 0.1 cm. The results are shown in Figure 1.

Table III: Volume Shape Factors

| Lithological Class | $\alpha_V$ |
|---|---|
| Turmaline Brec. | 0.29 |
| Other Breccias | 0.31 |
| Porph. Dykes | 0.53 |
| Dac. Diatreme | 0.47 |
| Granodiorites | 0.35 |
| Andesite | 0.44 |
| Riol. Diatreme | 0.34 |

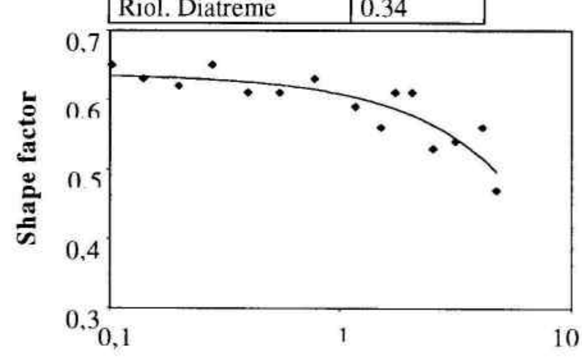


Figure 1. Volume Shape Factor vs. Particle Size

These results show that the characterization of an ore sample in terms of its shape factor, can not be performed without taking in account that it will depend on its lithological composition and on its particle size.

Since the shape factor is inversely proportional to the cubic diameter of the particles and this diameter is calculated as the geometric average between class size limits, any error in defining these limits will produce big errors in the resulting shape factors. Accordingly, the amplitude of each class size should be as narrow as possible.

## CONCLUSIONS

A new procedure to transform areal to weight distribution has been proposed. This method has been developed from the well-known transformation from surface to weight distribution and from the Heywood shape factors equations.

The procedure has been successfully tested using samples from seven different lithologies present in the Andina copper ore. Good results have been obtained, either with direct measurements of several rocks from each lithological class or with image analysis techniques over a composite disposed randomly on a multi layer arrangement. In both cases errors smaller than 3% are obtained.

The variability in the volume shape factor due to lithology, particle size and amplitude of the class size has been analyzed. The experimental results show that the volume shape factor of an ore sample will depend on its lithological composition and on its particle size. With respect to the class size the need for an amplitude of each class size as narrow as possible has been established.

Finally, it is important to highlight that the validity of the Heywood approach has been proved in terms of its general form. However, it should not be used with fixed parameters as the constants 1,57 and Kg of the surface shape factor equation, but using the lithological dependent constants $C_{1j}$ and $C_{2j}$.

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